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Brane localized curvature for warped gravitons.*

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Abstract: We study the effects of including brane localized curvature terms in the Randall-Sundrum (RS) model of the hierarchy. This leads to the existence of brane localized kinetic terms for the graviton. Such terms can be induced by brane and bulk quantum effects as well as Higgs-curvature mixing on the brane. We derive the modified spectrum of Kaluza-Klein (KK) gravitons and their couplings to 4-dimensional fields in the presence of these terms. We find that the masses and couplings of the KK gravitons have considerable dependence on the size of the brane localized terms; the weak-scale phenomenology of the model is consequently modified. In particular, the weak-scale spin-2 graviton resonances which generically appear in the RS model may be significantly lighter than previously assumed. However, they may avoid detection as their widths may be too narrow to be observable at colliders. In the contact interaction limit, for a certain range of parameters, the experimental reach for the scale of the theory is independent of the size of the boundary terms.

Keywords: Extra Large Dimensions, Beyond Standard Model.

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1. Introduction

The Randall-Sundrum (RS) model [1] provides a natural geometric picture for the hierarchy between the weak scale and the apparent gravitational scale, $\overline{M}_{Pl}$. In this model, two 3-branes truncate a 5-dimensional curved spacetime. In the original RS construction, the Standard Model (SM) fields reside on the SM (or TeV) brane where a scale, which is exponentially smaller than the fundamental 5-d scale is induced by the geometry. Gravity is localized on the second brane, known as the Planck brane. This exponential hierarchy is controlled by the distance r_c between the two branes and can generate the weak scale on the SM brane. The physics of the graviton Kaluza-Klein (KK) modes is governed by the same scale. It has been shown that r_c can be stabilized by bulk scalar fields [2], resulting in the appearance of a single weak scale radion field.

The most generic and distinctive signature of this model [1, 3] is the presence of TeV-scale spin-2 graviton KK resonances at colliders. It is important to examine whether modifications arising in a more realistic RS scenario could affect the KK graviton phenomenology. One such modification is the introduction of brane curvature terms for the graviton. These terms respect all 4-d symmetries, and are expected to be present in a 4-d effective theory, leading to Brane Localized Kinetic Terms (BLKT's) for the graviton. It has been argued that brane quantum effects can generate BLKT's [4, 5], and that these terms are required as brane counter terms for bulk quantum effects [6]. In addition, since the Higgs field is assumed to reside on the SM brane, a Higgs-curvature mixing term can be included in the brane action. Such a term is allowed by all 4-d symmetries and leads to Higgs-radion mixing with important phenomenological consequences [7, 8]. We observe that the same Higgs-curvature mixing term can also induce a BLKT for the graviton on the SM brane. Thus, there are many good theoretical reasons for assuming the presence of such terms for gravitons.

The effects of BLKT's on graviton physics have been studied for flat spacetimes [4, 9] and result in novel features. Boundary terms have also been shown to significantly modify the KK phenomenology of bulk gauge fields in flat [10] and warped geometries [11, 12].

For the case of an alternative RS model with only one brane and an uncompactified 5^{th} dimension, the gravitational and cosmological effects of these terms have been analyzed [13]. However, the case of the compactified RS model, which is relevant for weak scale physics, in principle involves graviton boundary terms on both branes and has yet to be examined. We study this case here and investigate the effects on the phenomenology of the weak scale KK gravitons in the presence of localized curvature terms.

We derive the modified graviton KK spectrum and couplings and show that a significant dependence on the size of such terms emerges. We illustrate that this leads to considerable changes in the low energy 4-d phenomenology of the RS model. These changes could have important implications for future searches and the potential discovery of model signatures. In addition, we derive bounds on the magnitude of the localized curvature terms on the Planck and TeV branes, by requiring the absence of graviton and radion ghost states.

The outline of this paper is as follows. In the next section, we derive the necessary formalism. Our numerical analysis is presented in section 3 and section 4 contains some concluding remarks.

2. Formalism

Here, we present our formalism and notation. We work in the RS background, as given by the following metric

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2\,, \tag{2.1}$$

where $\sigma = kr_c|\phi|$, k is the curvature scale of order $\overline{M}_{Pl}$, r_c is the radius of compactification, and $\phi \in [-\pi, \pi]$. The extra dimension is here compactified on an S^1/Z_2 orbifold. The gravitational action S_G, augmented by brane localized curvature terms on the Planck and TeV branes, is assumed to be given by

$$S_G = \frac{M_5^3}{4}\int d^4x \int r_c d\phi \sqrt{-G}\,\Big\{R^{(5)} + [g_0\,\delta(\phi) + g_\pi\,\delta(\phi-\pi)]\,R^{(4)} + \cdots\Big\}\,, \tag{2.2}$$

where M_5 is the 5-d reduced Planck mass, $G \equiv \det(G_{\mu\nu})$ and $g_{0,\pi}$ are numerical coefficients which are treated as phenomenological inputs in our calculations; one may expect $g_{0,\pi} \sim 1$ based on naturalness aguments. Here, $R^{(5)}$ denotes the 5-d gravitational curvature and $R^{(4)}$ is the 4-d curvature which is constructed only out of 4-d Minkowski metric components and derivatives. The terms proportional to $g_{0,\pi}$ represent the only modifications to the original RS proposal [1] that we study in this work. Note that we have assumed that the form of the brane terms are exactly the same as the bulk term except that they are calculated with the relevant 4-d metrics. In principle, one can imagine brane terms which are quadratic or even higher order in R, but these terms are small and would violate the philosophy of the original model where classical general relativity is applicable. As we will see below, brane terms of this type only alter the detailed structure while retaining the general nature of the usual RS solutions.

We will consider metric perturbations of the form

$$G_{\mu\nu} = e^{-2\sigma}(\eta_{\mu\nu} + \kappa_5 h_{\mu\nu})\,, \tag{2.3}$$

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with $\kappa_5 = 2M_5^{-3/2}$; $h_{\mu\nu}$ is the 5-d graviton field. The transverse-traceless gauge will be implemented in our calculations; this corresponds to taking $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$. We use a KK expansion for $h_{\mu\nu}$ of the form

$$h_{\mu\nu}(x,\phi) = \sum_n h^{(n)}_{\mu\nu}(x)\frac{\chi^{(n)}(\phi)}{\sqrt{r_c}}\,, \tag{2.4}$$

where $h^{(n)}_{\mu\nu}(x)$ are the KK modes and $\chi^{(n)}(\phi)$ are the wavefunctions along the extra dimension. With this expansion, eqs. (2.2) and (2.3) imply the following equation of motion

$$\frac{d}{d\phi}\left(e^{-4\sigma}\frac{d}{d\phi}\chi^{(n)}\right) + \left[1 + g_0\,\delta(\phi) + g_\pi\,\delta(\phi-\pi)\right]e^{-2\sigma}r_c^2\,m_n^2\,\chi^{(n)} = 0\,, \tag{2.5}$$

where m_n is the mass of the n^{th} KK mode. Away from the boundaries at $\phi = 0, \pi$, for $n \geq 1$, we have [3]

$$\chi^{(n)}(\phi) = \frac{e^{2\sigma}}{N_n}\,\zeta_2(z_n)\,, \tag{2.6}$$

where N_n are normalization constants, $\zeta_q(z_n) \equiv J_q(z_n) + \alpha_n\,Y_q(z_n)$, with $z_n(\phi) \equiv (m_n/k)e^\sigma$, and α_n are numerical coefficients to be fixed below; J_q and Y_q are Bessel functions of order q.

Let $\varepsilon_n \equiv z_n(0)$ and $x_n \equiv z_n(\pi)$. Integrating eq. (2.5) around $\phi = 0, \pi$ yields

$$\zeta_1(\varepsilon_n) + \gamma_0\,\varepsilon_n\zeta_2(\varepsilon_n) = 0 \tag{2.7}$$

and

$$\zeta_1(x_n) - \gamma_\pi x_n\zeta_2(x_n) = 0\,, \tag{2.8}$$

respectively, with $\gamma_{0,\pi} \equiv g_{0,\pi}\,kr_c/2$. Addressing the hierarchy problem requires $kr_c \sim 10$, hence, we expect $|\gamma_{0,\pi}| \lesssim 10$. Eq. (2.7) yields

$$\alpha_n = -\frac{J_1(\varepsilon_n) + \gamma_0\varepsilon_n J_2(\varepsilon_n)}{Y_1(\varepsilon_n) + \gamma_0\varepsilon_n Y_2(\varepsilon_n)}\,. \tag{2.9}$$

Here, we note that for $\gamma_0 \neq -1/2$,[1] $\alpha_n \sim \varepsilon_n^2$, where $\varepsilon_n \sim 10^{-15}$ for the lowest lying KK modes. The roots of eq. (2.8) give the masses $m_n = x_n k e^{-kr_c\pi}$. Note that with α_n sufficiently small we can safely neglect the Y_q components of the eigenvalue equation. This implies that the masses of the KK graviton tower are γ_0 independent unlike what happens in the corresponding case with gauge bosons in the bulk. It can be shown that for $\gamma_\pi \gg 1$, an additional root $x^* \ll 1$ is obtained, where

$$x^* \approx 2\left(\gamma_\pi + \frac{1}{2}\right)^{-1/2}\,. \tag{2.10}$$

This implies that there is a light mode in the spectrum for $\gamma_\pi \gg 1$. We will show that this signals an instability in this background.

[1] Later we will show that $\gamma_0 = -1/2$ is not a physically allowed value.

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Next, we will fix the normalization N_n of the KK modes by requiring

$$\int d\phi\, e^{-2\sigma}[1 + g_0\,\delta(\phi) + g_\pi\,\delta(\phi - \pi)]\,\chi^{(n)\,2} = 1\,. \tag{2.11}$$

For $n = 0$, this equation yields the zero mode wavefunction

$$\chi^{(0)} = \left(\frac{kr_c}{1 + 2\gamma_0}\right)^{1/2}, \tag{2.12}$$

which demonstrates that we must have $\gamma_0 > -1/2$ in order for the zero mode to be a physical state. For $n \geq 1$, eq. (2.11) gives

$$N_n^2 = \frac{e^{2kr_c\pi}\zeta_2^2(x_n)}{kr_c}\,(1 + \gamma_\pi^2 x_n^2 - 2\gamma_\pi)\,, \tag{2.13}$$

where terms of order $e^{-2kr_c\pi}$ have been ignored. In the limit that we can neglect the α_n, this form implies that the graviton wavefunctions are γ_0 independent. Note that this normalization factor remains positive for all values of γ_π so that this parameter is not constrained by these considerations.

We can now obtain the couplings of the KK gravitons to the energy momentum tensor $T_{\mu\nu}$ of the fields residing on the TeV brane at $\phi = \pi$. To do this, first we note that the action (2.2) yields

$$\overline{M}_{Pl}^2 = \frac{M_5^3}{k}(1 + 2\gamma_0)\,, \tag{2.14}$$

where $\overline{M}_{Pl}$ is the 4-d reduced Planck mass and terms of order $e^{-2kr_c\pi}$ have been ignored. The lagrangian for the interactions of $h_{\mu\nu}$ and the 4-d fields on the TeV brane is given by

$$\mathcal{L} = \frac{1}{M_5^{3/2}} h_{\mu\nu}(x, \pi) T^{\mu\nu}(x)\,. \tag{2.15}$$

Eqs. (2.6), (2.12), (2.13), and (2.14) then yield

$$\mathcal{L} = \frac{1}{\overline{M}_{Pl}} h_{\mu\nu}^{(0)}(x) T^{\mu\nu}(x) + \frac{1}{\Lambda_\pi} T^{\mu\nu}(x) \sum_{n=1}^{\infty} \lambda_n h_{\mu\nu}^{(n)}(x)\,, \tag{2.16}$$

with $\Lambda_\pi \equiv \overline{M}_{Pl} e^{-kr_c\pi}$ as usual and

$$\lambda_n \equiv \left(\frac{1 + 2\gamma_0}{1 + \gamma_\pi^2 x_n^2 - 2\gamma_\pi}\right)^{1/2}. \tag{2.17}$$

As expected, the zero mode coupling is $\overline{M}_{Pl}^{-1}$ and for $\gamma_0, \gamma_\pi \to 0$, the original RS model coupling for the non-zero mode KK states, Λ_π^{-1} [3], is retrieved. Choosing $\gamma_\pi \neq 0$ renders the KK couplings mode dependent, through λ_n. We see that for $\gamma_\pi x_n \gg 1$ these couplings have suppressions $\sim (\gamma_\pi x_n)^{-1}$.

Before continuing, we consider if there are any further constraints which can be imposed upon the parameters $\gamma_{0,\pi}$. Though here we are concerned with the effect of brane

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terms on the masses and couplings of the KK gravitons, the RS metric yields another excitation, the radion, whose properties may also be modified by such terms. Using the results of Csaki et al. [8], for example, the effects of the brane terms on the radion [14] can be determined since they are similar in form to those which induce Higgs-radion mixing. A short analysis shows that the brane terms lead to a multiplicative correction to the usual radion kinetic term, in the absence of the Higgs field, by a factor of $1-\gamma_\pi$; the corresponding γ_0-dependent contribution is found to be suppressed by powers of the exponential warp factor and we will ignore it in the following discussion. To avoid a radion ghost this implies that $\gamma_\pi \leq 1$, suggesting that negative or small positive values of γ_π are allowed. This multiplicative factor can then be absorbed by a suitable field rescaling so that the radion field becomes canonically normalized. However, once the Higgs-radion mixing term is included, this bound becomes a bit more complex since mixing and brane term contributions can compensate for one another. *If* the brane localized curvature for gravitons arises from solely the same source as Higgs-radion mixing [8, 14] then we find that a lower bound of $\gamma_\pi \gtrsim -1$ is also required. Of course, such brane terms can arise from many other sources so that smaller values of γ_π are certainly possible. Naturalness suggests that $\gamma_\pi \geq -10$ as discussed above and we will assume this soft bound in our discussion below.

3. Analysis

The first step in our analysis is to examine how the mass spectrum of the graviton KK modes is influenced by the presence of the brane kinetic terms parameterized via $\gamma_{0,\pi}$. As was noted above, both the root equation and the wave functions for the graviton KK modes in the extra dimension are found to be independent of γ_0 when powers of the warp factor, $e^{-\pi k r_c}$, are neglected; this implies that the x_n for the range of interest to us are essentially γ_0 independent. Recall that the masses of the KK states are now given by $m_m = x_n(\gamma_\pi) k e^{-k r_c \pi}$. Figure 1 displays these roots as a functions γ_π. (For purposes of comparison we note that the first root in the usual RS model without brane terms is given by $x_1 \simeq 3.83$.) For all KK modes, other than the case $n = 1$, the x_n are essentially constant away from the transition region near $\gamma_\pi = 0$. As we see from the root equation above, once $|\gamma_\pi|$ gets large the x_n are essentially just the roots of the equation $J_2(x_n) = 0$ which accounts for their mostly γ_π independent behaviour. The root x_1 behaves somewhat differently and scales as $\sim 2/\sqrt{\gamma_\pi}$ for large positive γ_π as seen from eq. (2.10) above. Note that this causes the ratio of the first two roots x_2/x_1 to grow substantially in the large and positive γ_π region.

Once the x_n are known the effective couplings of the KK modes to SM matter on the TeV brane can be immediately calculated and are shown in figure 2 for the case $\gamma_0 = 0$; for other values of γ_0 these results are scaled by a factor of $\sqrt{1+2\gamma_0}$. Except for the lowest KK mode when $\gamma_\pi > 0$, the couplings of all KK modes are observed to fall off drastically as the magnitude of γ_π increases, demonstrating the $1/|x_n \gamma_\pi|$ behavior, as expected. Clearly, the reduced couplings of these graviton KK modes will have a important impact on the ability to detect these states either directly or indirectly at colliders. This will be true in particular in the case of of positive γ_π, where as we saw above the second KK mode is

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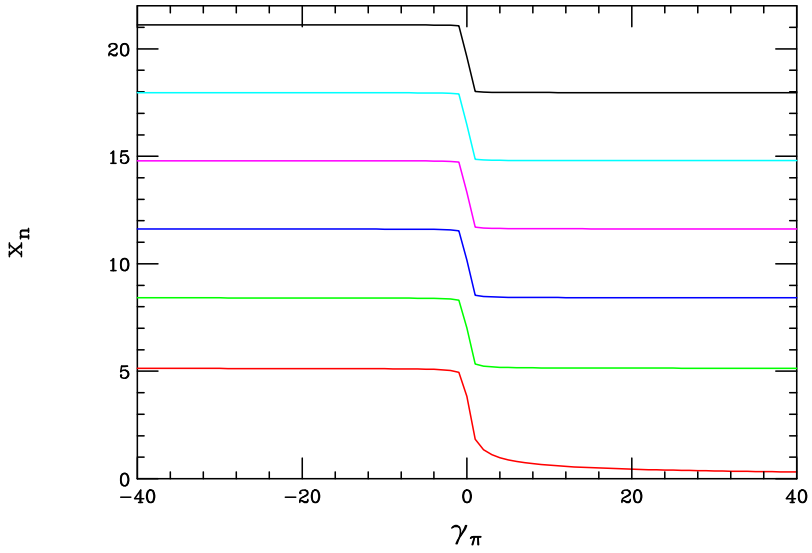

Figure 1: The first six roots x_n as functions of γ_π. These results are independent of γ_0 up to terms suppressed by warp factors. As discussed in the text, the region $\gamma_\pi < 1$ is required by the absence of radion ghosts taking the results of ref. [8]. This constraint is, however, model dependent.

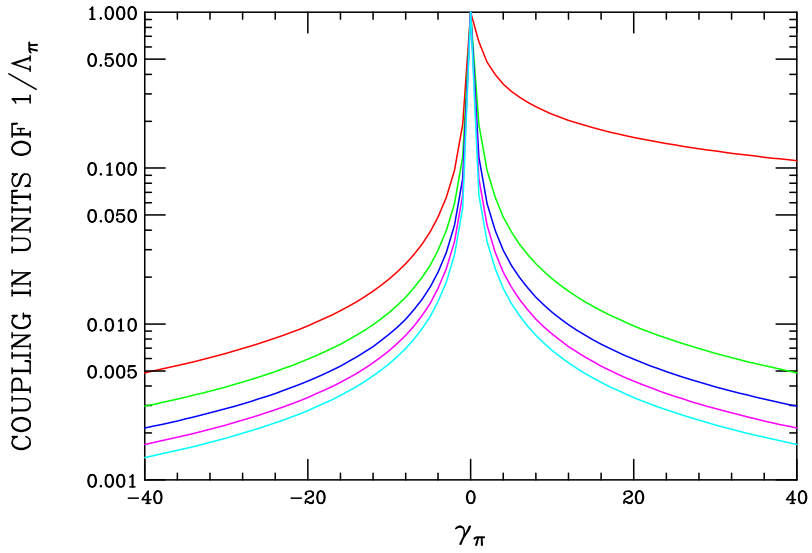

Figure 2: The couplings of the first five KK graviton tower modes as functions of γ_π assuming $\gamma_0 = 0$. The top curve corresponds to the lightest KK state with the more massive tower modes corresponding to the subsequently lower curves. The same restrictions on γ_π apply as in the previous figure.

significantly heavier than the first mode and has a much weaker coupling to the SM brane fields. However, we note that the absence of a radion ghost would exclude this extreme situation if we apply the analysis as given in ref. [8].

Perhaps the most important experimental signature for the RS model is the appearance of spin-2 KK resonances at colliders. As shown above, when γ_π is non-zero the masses and couplings of these KK states are altered from the usual expectations for

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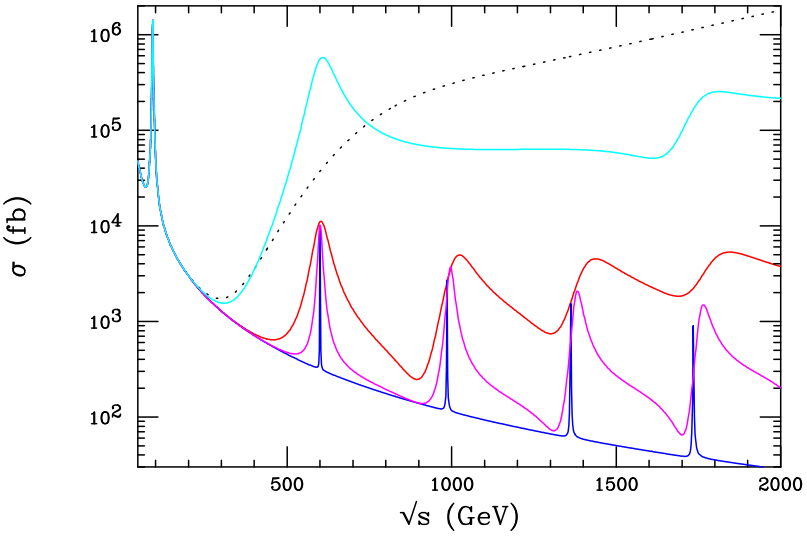

Figure 3: Cross section for $e^+e^- \to \mu^+\mu^-$ at a linear collider assuming $m_1 = 600\,\text{GeV}$ and $k/\overline{M}_{Pl} = 1$. The dotted curve is the standard RS result while from top to bottom on the right-hand side the curves correspond to $\gamma_\pi = 1(-1, -2, -10)$ represented by cyan(red, magenta, blue). In all cases $\gamma_0 = 0$ is assumed.

which the phenomenology has by now been well explored. Figures 3 and 4 show some of the possible modifications in the presence of the boundary terms to the production cross section and KK spectrum for the cases $k/\overline{M}_{Pl} = 1$ and 0.1, respectively, for the process $e^+e^- \to \mu^+\mu^-$ at a linear collider assuming the mass of the first KK resonance is 600 GeV. For the case of $k/\overline{M}_{Pl} = 1$ in the usual RS model, which corresponds to $\gamma_\pi = 0$ and is represented by the dotted curve in figure 3, the KK states are sufficiently strongly coupled to smear out the individual resonance structures and produce only a large shoulder in the cross section. For non-zero γ_π, there is a significant change in these expectations due to the reduced values of the couplings and the additional mass spectrum modifications. This is seen explicitly in these two figures. Note that the widths of these graviton resonances scale as $\Gamma_n/m_n \sim (x_n\lambda_n k/\overline{M}_{Pl})^2$ where λ_n is the relative coupling strength for the n-th KK mode calculated in eq. (2.17) and shown in figure 2. For positive values of γ_π the lowest KK mode is well separated in mass from the next more massive state. For $\gamma_\pi < 0$ the constructive interference between the resonances rapidly decreases resulting in a series of very narrow peaks once values of $\gamma_\pi \sim -10$ are reached. Since x_1 remains large in the negative γ_π case the peak height of the first KK state is similar to what might be expected in the conventional RS model without brane kinetic terms.

When $k/\overline{M}_{Pl} = 0.1$ the effects of the brane terms are in some sense much more severe as can be seen from figure 4. The usual resonances, the first few of which are reasonably narrow in the standard RS scheme in this case, are converted into narrow spikes some of which are quite small compared to the continuum background. For γ_π growing more and more negative the resonance structures beyond the first KK state become both smaller and more narrow and essentially disappear, fading into the continuum without extreme

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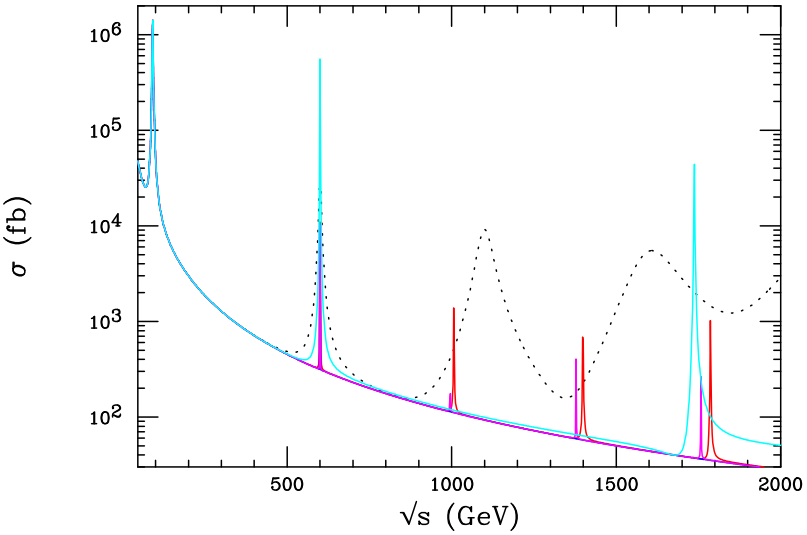

Figure 4: Same as the previous figure but now for $k/\overline{M}_{Pl} = 0.1$. The rapidly rising curve is the usual RS scenario prediction.

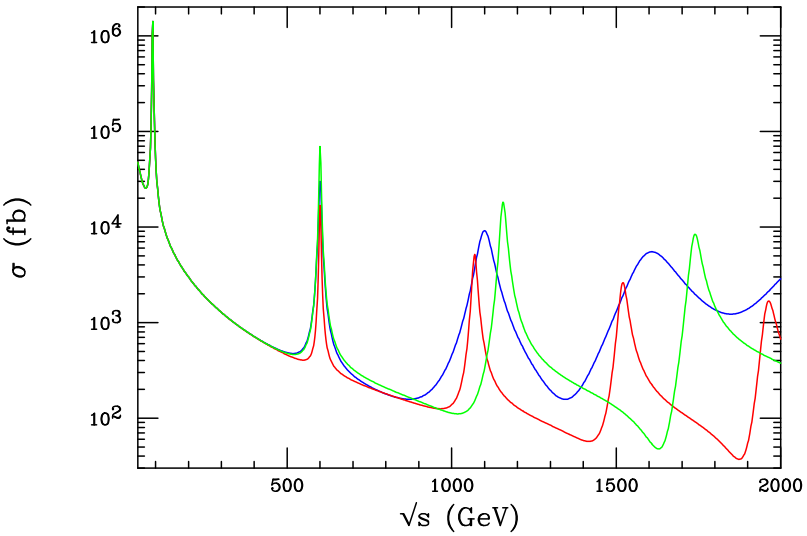

Figure 5: Same as the previous figure with $k/\overline{M}_{Pl} = 0.1$ but now for $\gamma_\pi = -0.2, 0$, and 0.2 corresponding to the red, blue and green curves, respectively, which are represented by the middle, top and bottom curves on the right-hand side.

resolution availability. With this or even smaller values of $k/\overline{M}_{Pl}$, it may be difficult to observe anything but the first KK graviton excitation over much of the range of γ_π. Thus it is very important to be able to observe and study in detail the first KK mode in order to elucidate the details of the model. The observation of only a single graviton KK resonance would thus be a potential signal for the presence of brane terms.

The shifts in the mass spectrum and couplings of the KK resonances induced by the brane kinetic terms are so large that even for values of $|\gamma_\pi| = 0.2$, there are substantial modifications to the production cross section. This is seen explicitly in figure 5.

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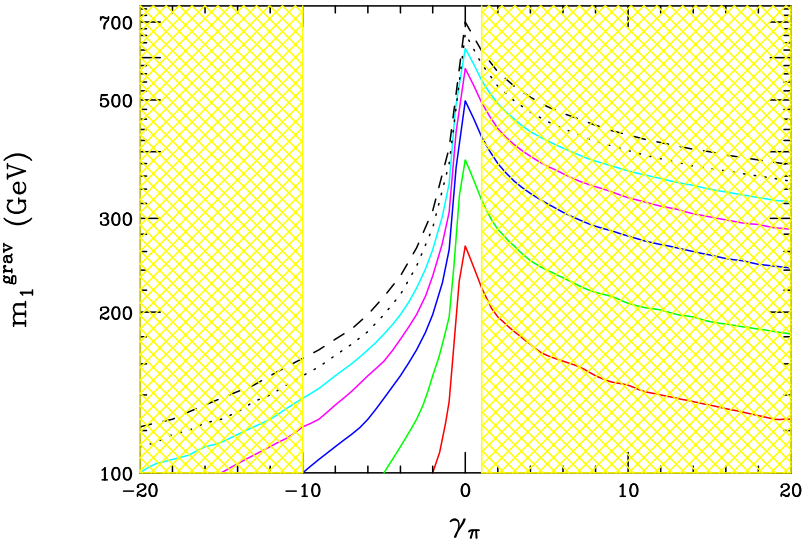

Figure 6: Search reach for the first graviton KK resonance employing the Drell-Yan channel at Tevatron Run I as a function of γ_π assuming $\gamma_0 = 0$. From bottom to top on the r.h.s. of the plot, the curves correspond to $k/\overline{M}_{Pl} = 0.01, 0.025, 0.05, 0.075, 0.10, 0.125$ and 0.15, respectively. The unshaded region is that allowed by naturalness considerations and the requirement of a ghost-free radion sector.

If the graviton KK modes are kinematically accessible they will most likely be first observed at a hadron collider. As discussed in our earlier work, the lack of observation of KK graviton resonances in, e.g., the Drell-Yan channel at the Tevatron already places a constraint on the RS model parameter space[3]. This analysis can be modified in order to examine the influence of brane kinetic terms for the graviton; the results are presented in figures 6, 7 and 8 keeping in mind that values of $k/\overline{M}_{Pl} \lesssim 0.1$ are theoretically preferred. These figures show the search reach for the first graviton KK resonance in Drell-Yan production as a function of γ_π for different values of $k/\overline{M}_{Pl}$.

The search reach from Run I of the Tevatron, which constitutes the present lower bound on the mass of the first KK graviton excitation, is shown in figure 6. Here we see that, as expected, the search reach degrades quite rapidly as we move away from the canonical value of $\gamma_\pi = 0$. Note that for negative values of γ_π it is still quite possible for the mass of this lightest state to be less than 200 GeV (provided this is not excluded by LEP searches for narrow resonances). For negative values of γ_π the corresponding bound on Λ_π can be approximately obtained from the m_1 constraint by dividing by $\simeq 5k/\overline{M}_{Pl}$.

Figure 7 shows that future running of the Tevatron during Run II can be useful in covering some of the large $|\gamma_\pi|$, small KK mass region. The higher energy and integrated luminosity will result in a reasonable increase in the overall parameter space coverage compared to Run I but, as can be seen from the figure, will still allow for the existence of very light KK gravitons.

Figure 8 shows the potential search reach for the first KK graviton resonance at the LHC. Even in this case one sees that there is a significant degradation in the reach away from $\gamma_\pi = 0$. Unlike in the standard RS scenario without brane terms we see that the

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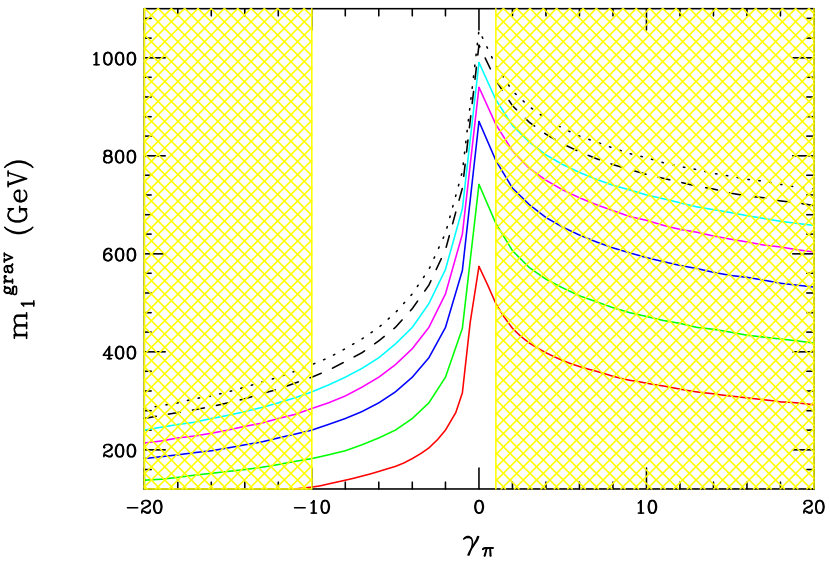

Figure 7: Same as the previous figure but now for Run II, $\sqrt{s} = 1.96\,\text{TeV}$, Tevatron assuming an integrated luminosity of 5 fb^{-1}.

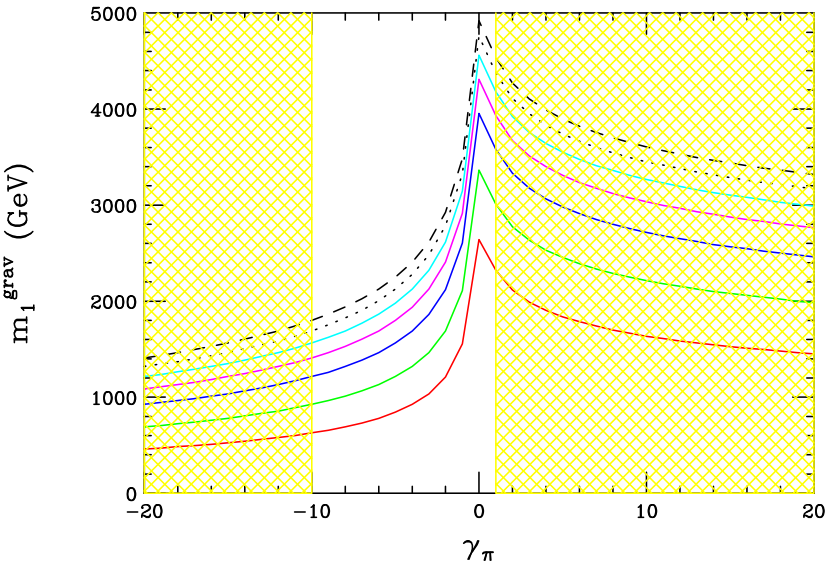

Figure 8: Same as the previous figure but now for the LHC with an integrated luminosity of 100 fb^{-1}.

LHC can no longer cover all of the interesting parameter space for this model due to the reduced coupling of the first KK resonance. For example we see from the figure that for large negative γ_π, a first KK graviton resonance with a mass of 600 GeV and $k/\overline{M}_{Pl} = 0.01$ may still miss detection — even at the LHC.

It is possible that the indirect effects of graviton KK tower exchange in particle pair production may be observed. This may be more important than direct graviton searches in some regions of parameter space. These indirect effects are similar to those of contact interactions, except that the graviton exchange operator is dimension-8. Graviton

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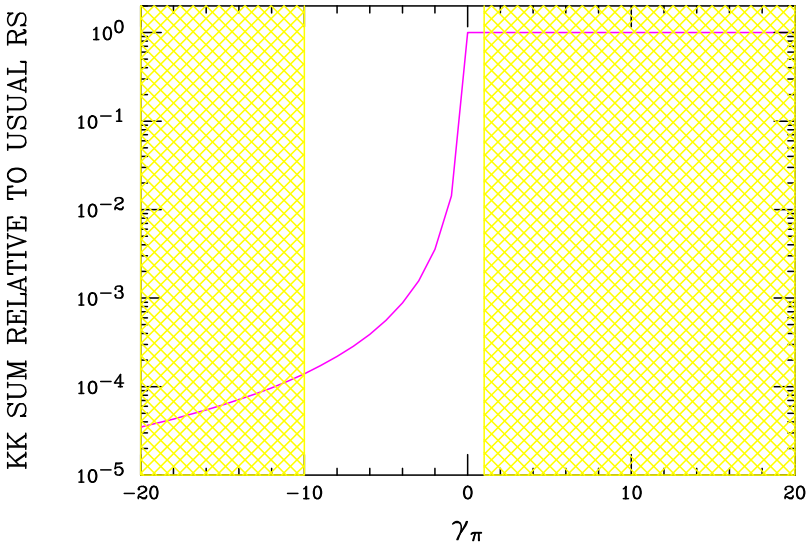

Figure 9: Sum over the graviton KK towers weighted by the appropriate coupling strength in comparison to that obtained in the standard RS model without brane kinetic terms as a function of γ_π assuming $\gamma_0 = 0$.

virtual effects have been computed for the case of large extra dimensions in ref. [15], where it was noted that the angular distributions of the final state particles may reveal the spin-2 nature of the indirect graviton exchange. Here, the 4-fermion matrix element is obtained from that for the large extra dimensions scenario with the replacement

$$\frac{\lambda}{M_H^4} \to \frac{\overline{M}_{Pl}^2}{8k^2\Lambda_\pi^4} \sum_n \frac{\lambda_n(\gamma_\pi,\gamma_0)^2}{x_n(\gamma_\pi)^2}\,, \tag{3.1}$$

in the limit of $m_n^2 \gg s$. In this model the sum rapidly converges. The coupling weighted sum over the KK tower exchanges differs from the conventional RS model by the ratio

$$\mathcal{R} = \frac{\sum_n \frac{\lambda_n(\gamma_\pi,\gamma_0)^2}{x_n(\gamma_\pi)^2}}{\sum_n \frac{\lambda_n(0)^2}{x_n(0)^2}}\,, \tag{3.2}$$

which is displayed in figure 9. We see from the figure that for positive values of γ_π the effects of the suppressed couplings and the modified roots exactly compensate each other. The standard RS predictions are thus maintained in this region. The contact interaction bounds on Λ_π obtained from the lepton pair invariant mass spectrum and forward-backward asymmetry in Drell-Yan production at the Tevatron and LHC are presented in figure 10 as a function of γ_π. The results from unpolarized (and polarized in the case of the Linear Collider) angular distributions, summing over e, μ, τ, c, b (and t if kinematically accessible) final states, as well as τ polarization distributions are shown in figure 11 for LEP II and the Linear Collider. From these figures we see that the constraints from 4-fermion interactions are rather weak for negative values of γ_π, even for the higher energy colliders. The searches from both direct graviton production at hadron

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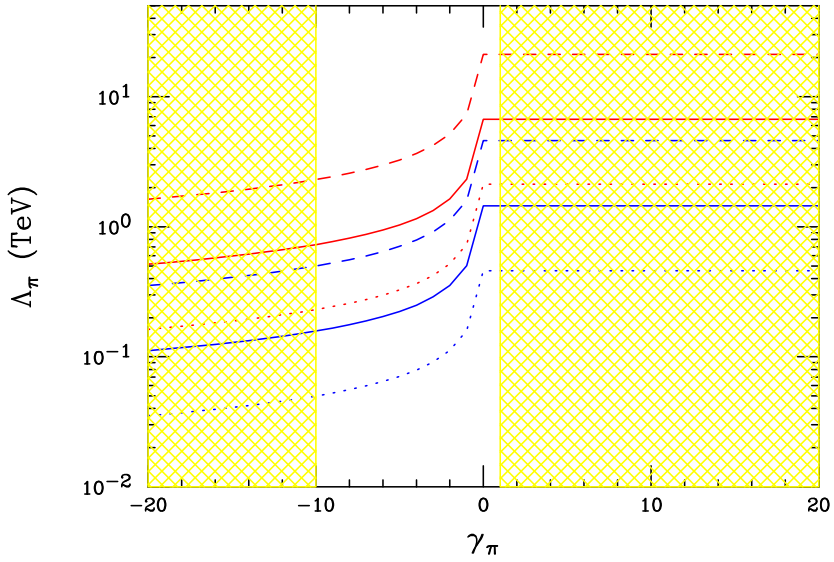

Figure 10: Contact interaction constraints on the scale Λ_π from the run II Tevatron with an integrated luminosity of 2 fb^{-1}(blue lower set of curves) and the LHC (red higher set of curves) with an integrated luminosity of 100 fb^{-1}. The dashed(solid, dotted) curves correspond to $k/\overline{M}_{Pl} = 0.01\,(0.1, 1)$, respectively. The shading is as in the previous figures.

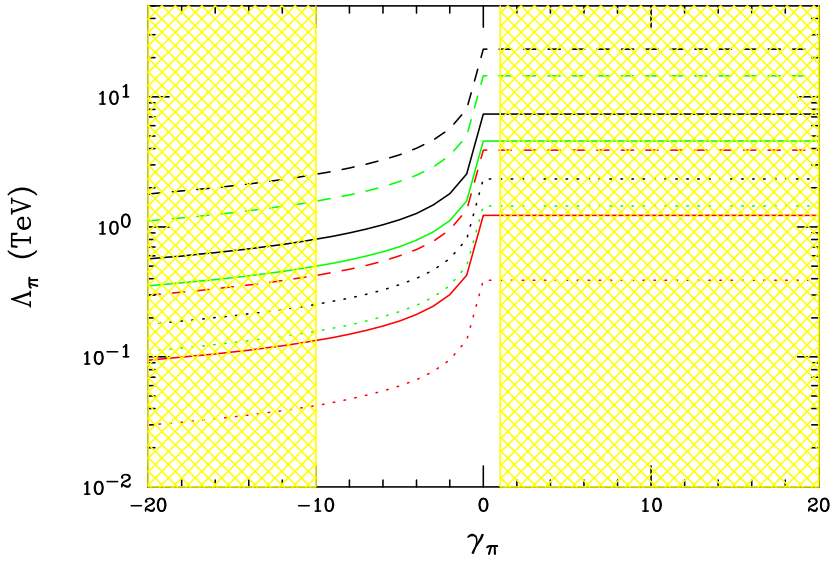

Figure 11: Same as the previous figure but now for LEPII(red lower set of curves), a 500 GeV Linear Collider (green, middle set), or a 1 TeV Linear Collider (black, higher set). The LC integrated luminosity is assumed to be 500 fb^{-1}.

colliders and the indirect exchange of graviton KK states in fermion pair production are thus difficult in this parameter space region and low mass graviton states may escape detection. It is possible that searches for narrow s-channel resonances in e^+e^- collisions or direct graviton production in $e^+e^- \to \gamma G$ may extend the discovery reach in this region.

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4. Conclusions

The most generic and distinctive signature of the RS model of the hierarchy is the appearance of weak scale, spin-2 KK resonances of the 5-d graviton. The discovery of such resonances at a collider will provide strong evidence for the RS picture. It is therefore important to understand how the spectrum and the couplings of these states could be affected by well-motivated theoretical modifications of the original proposal. The addition of brane localized curvature terms is such a modification. These terms yield contributions to the graviton BLKT's and can be generated from brane and bulk quantum effects. They can also arise at tree level due to Higgs-curvature mixing.

In this paper, we studied the consequences of adding brane curvature terms in the RS model. We derived the modified spectrum and couplings of the graviton KK states to the fields on the SM brane. The presence of localized curvature on the SM brane leads to important changes in the weak scale phenomenology. Requiring that the effective theory is free of KK and radion ghosts places lower and upper bounds on the coefficients γ_0 and γ_π of these terms on the Planck and SM branes, respectively.

We showed that, within the natural and allowed values of these coefficients, the variation in the shape and production cross sections of the resonances is significant. In particular, the KK resonances become very narrow for some regions of the parameter space and may be too narrow to be observed at colliders. We found that very light KK gravitons, of order 200-600 GeV may escape detection at the Tevatron and LHC. In the contact interaction limit, where the KK masses are too large for direct production, we observed that the collider reach for the SM scale $\Lambda_\pi \sim$ TeV varies considerably with $\gamma_\pi < 0$. However, for $\gamma_\pi > 0$, this reach remains constant. The cause of this behavior is the interplay of the modified masses and couplings of the KK states. In brief, we found that the modifications resulting from brane localized curvature terms in the RS model have interesting features and that they are important for future phenomenological studies and experimental searches.

Acknowledgments

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Erratum

The figure caption for figure 1 should read:

Figure 1: The first six roots x_n as functions of γ_π. These results are independent of γ_0 up to terms suppressed by warp factors. As discussed in the text, the region $\gamma_\pi < 1$ is required by the absence of radion ghosts taking the results of ref. [8].

Figures 3, 4, and 5 should be replaced by:

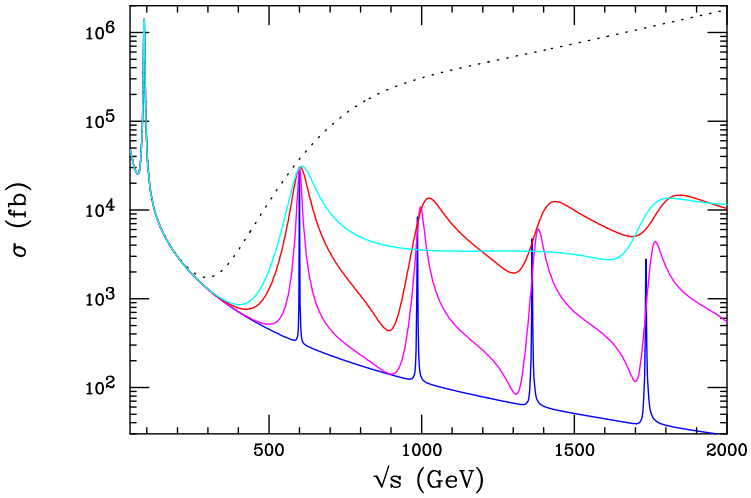

Figure 3': Cross section for $e^+e^- \to \mu^+\mu^-$ at a linear collider assuming $m_1 = 600$ GeV and $k/\overline{M}_{Pl} = 1$. The dotted curve is the standard RS result while from top to bottom on the right-hand side the curves correspond to $\gamma_\pi = 1(-1, -2, -10)$ represented by cyan(red, magenta, blue). In all cases $\gamma_0 = 0$ is assumed.

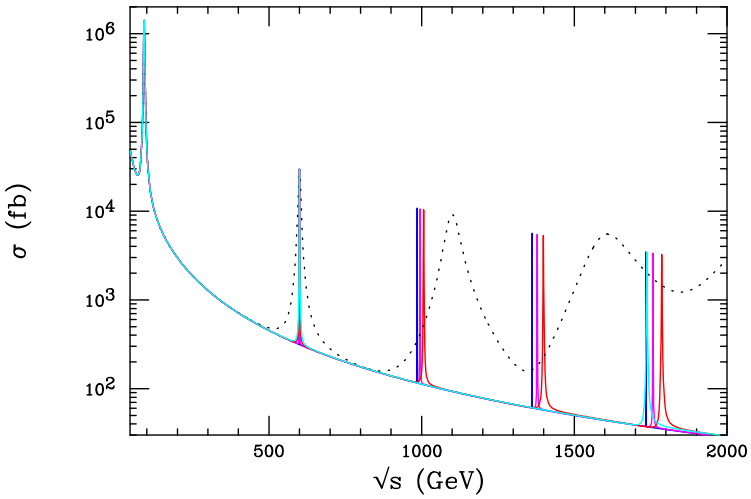

Figure 4': Same as the previous figure but now for $k/\overline{M}_{Pl} = 0.1$. The rapidly rising curve is the usual RS scenario prediction. The resonances for the case $\gamma_\pi = -10$ have essentially become spikes here due to the resolution in our plotting routine.

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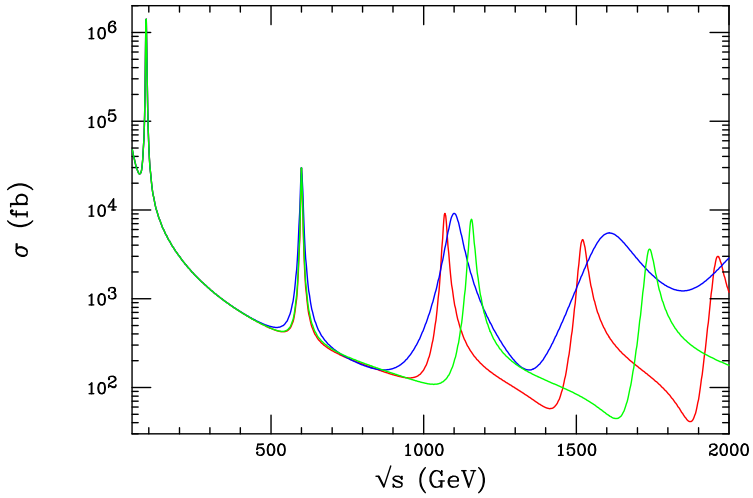

Figure 5': Same as the previous figure with $k/\overline{M}_{Pl} = 0.1$ but now for $\gamma_\pi = -0.2, 0$, and 0.2 corresponding to the red, blue and green curves, respectively, which are represented by the middle, top and bottom curves on the right-hand side.

The qualitative nature of the figures has not changed. However, some of the detailed quantitative discussion of the figures has been modified. The text starting with line 14 on page 7, i.e., 'For $\gamma_\pi < 0 \ldots$' until and including line 5 on page 8, i.e., 'potential signal for presence of brane terms.' should be replaced with:

For $\gamma_\pi < 0$ the strong constructive interference between the resonances rapidly vanishes and individual peaks become observable. As γ_π decreases further this results in a series of very narrow, spike-like peaks once values of ~ -10 are reached.

When $k/\overline{M}_{Pl} = 0.1$ the effects of the brane terms are in some sense much more severe as can be seen from figure 4' due to the further reduction in the resonance widths by a factor of 100. The usual resonances, the first few of which are reasonably narrow in the standard RS scheme in this case, are converted into narrow spikes some if not all of which will yield event rates which are quite small compared to the continuum background in any given mass bin. For γ_π positive and of order unity the unique feature here is the large mass gap between the first and second resonances. For γ_π negative the resonance structures are also narrow but with a smaller mass gap. With this or even smaller values of $k/\overline{M}_{Pl}$, it may be difficult to observe anything, even the first KK graviton excitation, over much of the range of γ_π unless use is made of radiative returns or one is lucky choosing the collider center of mass energy in performing a scan. It is clear from our discussion that it is important to see at least two of these resonance peaks to uniquely identify the model and extract out the relevant parameters.

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